

# Jordan algebra and statistical inference in linear mixed models

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## Abstract

In the presentation we consider the well known mixed linear model and the problems of estimation and testing hypotheses both for fixed parameters and variance components. Necessary and sufficient conditions for existence of best linear unbiased estimators BLUE's for linear functions of fixed effect and best quadratic unbiased estimators BQUE's for variance components of random effects will be given. It will be shown, that Jordan Algebras play very important role in such characterization. Under normality assumption those conditions are equivalent for existence of BUE's. Moreover, in the case of normality assumptions, we can get some optimal tests based on those estimators and it can be shown that the some tests are based on statistics as a functions of BLUE's and/or BQUE's which are F distributed. Also, in this case the assumptions on the covariance structure is very important and Jordan Algebras play main role. We would like to refer readers to the following papers and to the references of those papers.

## References:

- Fonseca, M., J. T. Mexia and R. Zmyślony (2006). Binary operations on Jordan algebras and orthogonal normal models. *Linear Algebra Appl.* 417, 75-97.
- Fonseca, M., J. T. Mexia and R. Zmyślony (2002). Exact distributions for the generalized F tests. *Discuss. Math. Probab. Stat.* 26, 37-51.
- Gnot, S., W. Klonecki and R. Zmyślony (1978). Linear spaces and the theory of best linear unbiased estimation. *Bull. Pol. Acad. Sci. Math.* 26, 69-72.
- Gnot, S., W. Klonecki and R. Zmyślony (1978). Best linear unbiased estimation, a coordinate free approach. *Probab. Math. Statist.* 1, 1-13.
- Michalski, A. and R. Zmyślony (1996). Testing hypothesis for variance components in mixed linear models. *Statistics* 27, 297-310.
- Michalski, A. and R. Zmyślony (1996). Testing hypothesis for linear functions of parameters in mixed linear models. *Tatra Mt. Math. Publ.* 17, 103-110.

- Jordan, P., J. von Neumann and E.P. Wigner (1934). On an algebraic generalization of the quantum mechanical formalism, *Ann. Math. II Ser.* 35, 29-64.
- Seely, J. (1971). Quadratic subspaces and completeness. *Ann. Math. Stat.* 42, 1735-1748.
- Zmyślony, R. (1980). A characterization of best linear unbiased estimators in the general linear model. *Mathematical Statistics and Probability Theory*, Proc. 6th Int. Conf. Wisa, Poland, 1978, Lecture Notes in Statist., 365-373.
- Zmyślony, R. (1980). Completeness for a family of normal distributions. *Mathematical Statistics*, Banach Center Publ. 6, 355-357