An illustrated philatelic introduction to magic square matrices

George P. H. Styan[1] **and Götz Trenkler**[2]

[1]*McGill University, Montréal (Québec), Canada*
[2]*Universität Dortmund, Germany*

Abstract

In this talk we focus on magic square matrices and associated postage stamps. As noted by Barnard (1888), "The construction of magic squares has been practiced earlier than the period of authentic history and it has preoccupied the attention of the curious in every age, among them men of high scientific eminence." The magic square associated with the matrix

$$L = \begin{pmatrix} 4 & 9 & 2 \\ 3 & 5 & 7 \\ 8 & 1 & 6 \end{pmatrix}$$

is, according to legend, over 4000 years old and is known as the *luoshu* or "Luo River Writing" (Needham 1970, page 56; Swetz 2003, page 2). It was apparently first considered by the Chinese engineer-emperor Yü the Great (*fl.* 21st century BC) in connection with the arrangement of dots on the shell of a turtle from the River Luo, a tributary of the Yellow River.

While magic squares have a very long history, postage stamps are less than 200 years old; the first postage stamp, the "Penny Black", was issued by Great Britain in May 1840 (Arthur Cayley introduced matrices in 1857). Since 1840 many thousands of stamps have been issued, catalogued and collected. As Schaaf (1978, page xiiii) observed, stamp collectors and philatelists deal with a rich and fascinating world which is "a mirror of civilization". Schaaf's *Adventure in Postage Stamps* considered "the impact of mathematics and science on society". As for science, the nuclear physicist Ernest Rutherford (1871–1937) said "All science is either physics or stamp collecting" (Blackett 1962, page 108; Wilson 2001, book title page)—Rutherford won the Nobel Prize in chemistry in 1908.

While we have not found any matrices actually depicted on postage stamps, we have found three stamps which depict magic squares; the associated matrices are:

$$A = \begin{pmatrix} 16 & 3 & 2 & 13 \\ 5 & 10 & 11 & 8 \\ 9 & 6 & 7 & 12 \\ 4 & 15 & 14 & 1 \end{pmatrix}, \qquad B = \begin{pmatrix} 52 & 61 & 4 & 13 & 20 & 29 & 36 & 45 \\ 14 & 3 & 62 & 51 & 46 & 35 & 30 & 19 \\ 53 & 60 & 5 & 12 & 21 & 28 & 37 & 44 \\ 11 & 6 & 59 & 54 & 43 & 38 & 27 & 22 \\ 55 & 58 & 7 & 10 & 23 & 26 & 39 & 42 \\ 9 & 8 & 57 & 56 & 41 & 40 & 25 & 24 \\ 50 & 63 & 2 & 15 & 18 & 31 & 34 & 47 \\ 16 & 1 & 64 & 49 & 48 & 33 & 32 & 17 \end{pmatrix}.$$

The magic square associated with the matrix A appears in Albrecht Dürer's engraving *Melencolia I*, which is depicted on a 1986 stamp issued by Aitutaki–Cook Islands and on a 1978 stamp from Mongolia (Wilson 2001, frontispiece). The magic square associated with the matrix B was published in 1769 by Benjamin Franklin (1706–1790) and is shown on a stamp issued by the USA in 2006.

In this talk we also look philatelically at several other magic squares, including two to be found in Europe: in the Sagrada Família basilica in Barcelona and in the Villa Albani in Rome, and three in India: in the Gwalior Fort in Madhya Pradesh, in a Hindu temple in Dudhai (Jhansi District), and in the Jain Parshvanath Temple in Khajuraho.

Keywords

Arthur Cayley (1821–1895), Albrecht Dürer (1471–1528), Benjamin Franklin (1706–1790), Frederick Augustus Porter Barnard (1809–1889), Ernest Rutherford (1871–1937); Franklin squares, History of mathematics, *Luoshu*, Magic squares, Philately, Postage stamps.

References:

Barnard, F.A.P. (1888). Theory of magic squares and of magic cubes. *Memoirs of the National Academy Sciences USA*, vol. IV, Sixth Memoir, pp. 207–270.

Blackett, P.M.S. (1962). Memories of Rutherford. In *Rutherford at Manchester* edited by J. B. Birks, London: Heywood, pp. 102–113.

Needham, Joseph, with the collaboration of Wang Ling (1970). *Science and Civilisation in China, Volume 3: Mathematics and the Science of the Heavens and Earth*, Reprint edition. Cambridge University Press. [Originally published: 1959.]

Schaaf, W.L. (1978). *Mathematics and Science: An Adventure in Postage Stamps.* Reston, Virginia: National Council for Teachers of Mathematics.

Swetz, F.J. (2002). *Legacy of the Luoshu: The 4,000 Year Search for the Meaning of the Magic Square of Order Three.* Chicago: Open Court.

Wilson, R.J. (2001). *Stamping Through Mathematics.* New York: Springer-Verlag.