A numerical study of planar and near-planar adjacency matrices used in geographical analysis*

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Abstract

Geographical analysis most frequently involves irreducible, non-negative (i.e., 0-1 binary) square adjacency matrices associated with planar or near-planar [i.e., a total number of 1s only slightly exceeding 6(n-2), where n is the number of nodes] graphs that can reach sizes for which n is at least 1,000,000. Eigenvalues of these matrices are needed for a variety of spatial statistical analysis purposes. Current computer technology and linear algebra theory support calculation of these eigenvalues for such a matrix as long as its size does not exceed approximately n = 10,000. Quantum physics encounters a similar problem, but for regular, random matrices, and as such, furnishes some useful guidelines for numerically studying the eigenfunctions of (near-)planar graph adjacency matrices that are expressed in terms of C-, S- and W-coding schemes. This paper begins by addressing the numerical preliminaries for sparse matrices of: confirming irreducibility, simple evaluations of the presence of 0 eigenvalues, and estimating the extreme eigenvalues. Next, based upon a database comprising more than seven dozen regular and irregular surface partitionings, most of which have been used in published spatial statistical analyses, both power law descriptions of eigenvalue distributions and time-series ARIMA descriptions of eigenvalue spacings are summarized.

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