A statistical interpretation of Krylov subspaces decomposition

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Abstract

Consider the standard linear model

$$Y = X\beta + \varepsilon.$$

where β the $p-$vectors of unknown parameters. To stabilize the OLS estimator when explanatory variables are highly correlated, PLS uses a Gram-Schmidt decomposition of the Krylov subspaces

$$K_q = \text{span}(s, S\,s, S^2\,s, ..., S^{q-1}\,s)$$

generated by $S = X'X$ and $s = X'Y$. We show how this decomposition can be obtained from an algorithm that iteratively maximizes the directional signal-to-noise ratio (SNR) applied to the least squares estimator under orthogonality constraints. The SNR on the direction given by $x \in R^p$ is defined by

$$\text{SNR} = \frac{|x'\,\widehat{\beta}^{ols}|}{\sigma\,\sqrt{x'\,S^{-1}x}},$$

and is related with optimal shrinkage factors that realize optimal trade-off between bias and variance.

Keywords

PLS regression, Krylov subspaces, Signal-to-noise ratio.

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