

Testing for structure in covariance matrices

The advantage of the use of conditionally independent tests in
building near-exact distributions for the test statistics

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Abstract

We show how several rather elaborate structures for covariance matrices may be tested by decomposing the null hypotheses under study in a number of 'partial' hypotheses which are conditionally independent. Using this technique we are able not only to construct quite easily the overall likelihood ratio test statistics but also to obtain, from the decomposition of the characteristic function of the statistics used to test the 'partial' hypotheses, several near-exact distributions for such overall test statistics. These distributions are of great importance given the intractability of the exact distributions and the non-existence of asymptotic distributions for most cases.

Among the tests studied are the tests of multisample sphericity, block sphericity and multisample block sphericity. As partial tests we use the test for independence of several sets of variates, the test of equality of several covariance matrices and the sphericity test.

Two measures based on the Berry-Esseen inequality and on the inversion formulas are considered to assess the quality of the near-exact distributions developed.

Keywords

Near-exact distributions, Likelihood ratio test, Asymptotic distributions, Sphericity, Block sphericity, Structure, Covariance matrices.

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